

## Chapter 16: Microeconomics

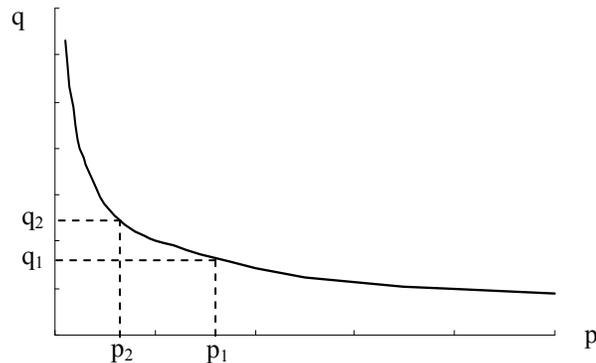
**Prerequisites:** Chapter 5

### 16.1 *The Notion of Elasticity*

In this section we will be making four key assumptions about demand for products and services.

- (1) Consumers maximize utility,
- (2) Consumers have full knowledge of all relevant market conditions,
- (3) Sellers maximize short-term profit and
- (4) Demand is infinitely divisible.

Imagine that we are looking at the relationship between price and quantity demanded. In the figure below, we have highlighted two price points,  $p_1$  and  $p_2$ , and the two corresponding demand points.



The elasticity represents the percentage change in the quantity demanded, which is represented on the y axis of the graph above, divided by the percentage change in the price demanded, which shows up on the x axis. We will be using the symbol  $e$  to refer to the elasticity. In that case we have

$$e = \frac{q_1 - q_2 / q_1}{p_1 - p_2 / p_1} = \frac{\Delta q / q}{\Delta p / p} = \frac{\Delta q}{\Delta p} \cdot \frac{q}{p}$$

If we assume that  $\Delta q$  and  $\Delta p$  can be made arbitrarily small, meaning that price and demand are both infinitely divisible, we can take the following step

$$\begin{aligned} e &= \frac{\Delta q}{\Delta p} \cdot \frac{q}{p} \\ &= \frac{dq}{dp} \cdot \frac{q}{p} \end{aligned} \tag{16.1}$$

where  $dq/dp$  is the derivative of  $q$  with respect to  $p$ . By time honored tradition, if  $-1 \leq e \leq 0$  we say that the demand is inelastic. On the other hand, if  $e < -1$  we say that demand is elastic.

In Chapter 5 we studied the linear model which in this context would be

$$q_i = \beta_0 + p_i \beta_1. \quad (16.2)$$

Dropping the subscript  $i$ , and since  $dq/dp = \beta_1$  (if you wish you can review Section 3.2), obviously under the linear model in Equation (16.2) it is then the case that

$$e = \beta_1 \frac{p}{q}.$$

Continuing to assume that the model of Equation (16.2) holds, we can now substitute  $\beta_0 + p\beta_1$  for  $q$  to give us

$$e = \beta_1 \frac{p}{\beta_0 + p\beta_1}.$$

A linear demand function creates a situation in which the elasticity depends on  $p$ . Now let's try (as we did in Section 7.6) a quadratic function,

$$q_i = \beta_0 + p_i \beta_1 + p_i^2 \beta_2.$$

Then, again dropping the subscript  $i$ ,  $dq/dp = \beta_1 + 2p\beta_2$  and therefore, according to Equation (16.1) we have

$$e = (\beta_1 + 2p\beta_2) \frac{p}{q}.$$

OK, now we are ready for the Cobb-Douglas function that models demand as

$$q_i = \beta_0 p_i^{\beta_1} \text{ or} \quad (16.3)$$

$$\ln q_i = \ln \beta_0 + \beta_1 \ln p_i. \quad (16.4)$$

Note that while the model is nonlinear, we can easily estimate it using OLS because it can be linearized by taking the log of the independent and dependent variables. When we estimate Equation (16.4) we get the same value of interest,  $\beta_1$ , that we see in Equation (16.3). The derivative is

$$\frac{dq}{dp} = \beta_1 \beta_0 p^{\beta_1 - 1}$$

so that

$$e = \beta_1 \beta_0 p^{\beta_1 - 1} \frac{p}{q}.$$

Again, we see  $q$  in the equation so we substitute the model in Equation (16.3) to get

$$e = \frac{\beta_1 \beta_0 p^{\beta_1 - 1} p^1}{\beta_0 p^{\beta_1}}.$$

Note that  $p$  has been written as  $p^1$  just so that we can use the rule of Equation (3.7) and end up with  $p^{\beta_1}$  in the numerator. Everything cancels, except for a single term and we have, for the Cobb-Douglas function,

$$e = \beta_1 \quad (16.5)$$

which shows us that the Cobb-Douglas is a constant elasticity model, meaning that the elasticity stays the same all along the x-axis of price.

### 16.2 *Optimizing the Pricing Decision*

In this section we are not going to assume either the Cobb-Douglas function of Equation (16.3) or any other particular demand function. Instead, we leave it that sales are a function of price, i.e.  $q = f(p)$ . But we are not optimizing demand, we are interested in optimizing profit which requires that we take costs into account. Lets say that costs are a function of quantity demanded, i. e.  $c = g(q)$ . In summary, we wish to make

$$\rho = \text{revenue} - \text{cost} = pq - g(q) \quad (16.6)$$

as large as possible. The breakeven point occurs when

$$pq = g(q)$$

$$p = \frac{g(q)}{q}$$

as revenue is equal to cost at that point. But we don't want to just break even, instead we want  $dp/p = 0$  as this would be the point at which the function is at an extreme point. We have

$$\frac{dp}{dp} = \frac{dp \cdot q}{dp} + \frac{dg(q)}{dp} = 0$$

since the sum of the derivatives are equal to the derivative of the sum [Equation (3.12)]. When the above equation is at zero,

$$\frac{dp \cdot q}{dp} = - \frac{dg(q)}{dp}$$

Rewriting the Equation (16.6), we see that

$$\rho = p \cdot f(p) - g[f(p)]$$

meaning that according to the chain rule [Equation (3.14)] the derivative is

$$\frac{d}{dp} p \cdot f(p) - g[f(p)] = f'(p) \cdot p + f(p) - g'[f(p)] \cdot f'(p)$$

### *References*

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